Substructural Weakest Preconditions in Fibrations

John Li, Ryan Doenges, Pedro Azevedo de Amorim

Weakest preconditions

- A modal alternative to Hoare logic that makes Hoare triples “first class” (cf. dynamic logic).

$$\{ P \}\ e\ \{ x.\, Q \}$$

$$P \subseteq \mathrm{wp}(e,\, x.\ Q)$$

$$P \subseteq \mathrm{wp}(e, x.\ Q)$$

$$\mathrm{wp}(e, x.\ Q) = \{\ s \mid e(s) = (s', x) \text{ and } s' \in Q(x)\ \}$$

$$\frac{\{\, P \,\}\; e\; \{\, x.\; Q \,\} \qquad \forall x.\; \{\, Q \,\}\; k\, x\; \{\, y.\; R \,\}}{\{\, P \,\}\; e \mathbin{>\!\!>\!\!=} k\; \{\, y.\; R \,\}}$$

$$\mathrm{wp}(e,\, x.\ \mathrm{wp}(k\, x,\, y.\ R)) \subseteq \mathrm{wp}(e >\!\!>\!= k,\, y.\ R)$$

Substructural weakest preconditions

- Modern wps are defined in separation logic, to support reasoning about resources (e.g., Iris).

- Defining these substructural wps is the primary challenge of building new program logics.

Iris

$$
\begin{aligned}
\mathsf{wp}\ e\ \{\Phi\} \triangleq\ & (e \in \mathit{Val} \wedge \dot{\Rrightarrow} \Phi(e)) \\
& \vee \Big(e \notin \mathit{Val} \wedge \forall \sigma.\ \boxed{\bullet\, \sigma}^{\gamma_{\mathrm{HEAP}}} -\!\!* \dot{\Rrightarrow} \big(\mathrm{red}(e, \sigma) \\
& \quad \wedge \triangleright \forall e_2, \sigma_2, \vec{e}_f.\ \big((e, \sigma) \rightarrow_{\mathsf{t}} (e_2, \sigma_2, \vec{e}_f)\big) -\!\!* \dot{\Rrightarrow} \\
& \qquad \big(\boxed{\bullet\, \sigma_2}^{\gamma_{\mathrm{HEAP}}} * \mathsf{wp}\ e_2\ \{\Phi\} * \mathop{\Huge *}_{e' \in \vec{e}_f} \mathsf{wp}\ e'\ \{v.\mathsf{True}\}\big)\big)\Big)
\end{aligned}
$$

$$
\ell \mapsto v \triangleq \boxed{\circ\, [\ell \leftarrow v]}^{\gamma_{\mathrm{HEAP}}}
$$

Lilac (probabilistic separation logic)

$$\gamma, D, \mathcal{P} \vDash \mathrm{wp}(M, X{:}A.Q) \quad \text{iff} \quad \begin{array}{l} \text{for all } \mathcal{P}_{\text{frame}} \text{ and } \mu \text{ with } \mathcal{P}_{\text{frame}} \bullet \mathcal{P} \sqsubseteq (\Sigma_\Omega, \mu) \\ \text{and all } D_{\text{ext}} : \mathrm{RV}\,[\![\Delta_{\text{ext}}]\!] \\ \text{there exists } X : \mathrm{RV}\,A \text{ and } \mathcal{P}' \text{ and } \mu' \text{ with } \mathcal{P}_{\text{frame}} \bullet \mathcal{P}' \sqsubseteq (\Sigma_\Omega, \mu') \\ \text{such that} \begin{pmatrix} \omega \leftarrow \mu; \\ v \leftarrow M(\gamma)(D(\omega)); \\ \text{ret } (D_{\text{ext}}(\omega), D(\omega), v) \end{pmatrix} = \begin{pmatrix} \omega \leftarrow \mu'; \\ \text{ret } (D_{\text{ext}}(\omega), D(\omega), X(\omega)) \end{pmatrix} \\ \text{and } \gamma, (D, X), \mathcal{P}' \vDash Q \end{array}$$

Borrowing

$$\mathsf{wp}\,(e)\,\{\hat{Q}\} \triangleq \forall\, \rho_f \# \rho.\ \exists\, \rho' \# \rho_f, \rho^+ \# (\rho_f \bullet \rho'), v.\\ (\llbracket \rho_f \bullet \rho \rrbracket, e) \to^* (\llbracket \rho_f \bullet \rho' \bullet \rho^+ \rrbracket, v) \wedge \rho \leftrightsquigarrow \rho' \bullet \rho^+ \wedge \rho^+|_{\mathsf{own}} = \varnothing \wedge \hat{Q}(v)(\rho')$$

$$\rho_1 \leftrightsquigarrow \rho_2 \triangleq \begin{cases} \forall\, \ell, \alpha, \overline{\beta}, v, \rho, \hat{P}. \\ ((\!|\rho_1|\!)(\ell) = \mathsf{mut}(\alpha, \_, \_, \hat{P}) \Leftrightarrow (\!|\rho_2|\!)(\ell) = \mathsf{mut}(\alpha, \_, \_, \hat{P})) \\ \wedge ((\!|\rho_1|\!)(\ell) = \mathsf{imm}(\overline{\beta}, v, \rho) \Leftrightarrow (\!|\rho_2|\!)(\ell) = \mathsf{mut}(\overline{\beta}, v, \rho)), & \checkmark \rho_1 \wedge \checkmark \rho_2 \end{cases}$$

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To learn more about this one, stick around for:

Sat 18 Oct

16:45 **15m** ☆ **From Linearity to Borrowing**

Talk Andrew Wagner Northeastern University, Olek Gierczak Northeastern University, Brianna Marshall Northeastern University, John Li Northeastern University, Amal Ahmed Northeastern University, USA

The dream

- wps for free: reliable methods for going from a description of a computational effect to a good substructural wp for it.
- Reuse: constructions for getting fancier wps from simpler ones (e.g., relational from unary, step-indexed from non).

The reality

- Good wps are found through trial and error.
- Expected properties are (re)checked by hand each time (e.g., the frame rule, the “bind lemma”, desired proof rules).
- No agreed-upon definition for “good substructural wp”.

Towards a categorical definition of substructural wp

Substructural weakest preconditions <u>in fibrations</u>

- Our substructural take on **Weakest preconditions in fibrations** Alejandro Aguirre[1], Shin-ya Katsumata[2,*] and Satoshi Kura[3] .

- We define “good substructural wp” to mean a *strong lifting* of a computational monad along a *bunched fibration*.

- Through this, we hope to make general facts about monad and liftings useful to the design of good substructural wps.

Background: Weakest preconditions in fibrations

Alejandro Aguirre[1], Shin-ya Katsumata[2,*] and Satoshi Kura[3]

- A model of predicate logic forms a functor $\mathrm{Prop} : \mathrm{Set}^{\mathrm{op}} \to \mathrm{HA}$.

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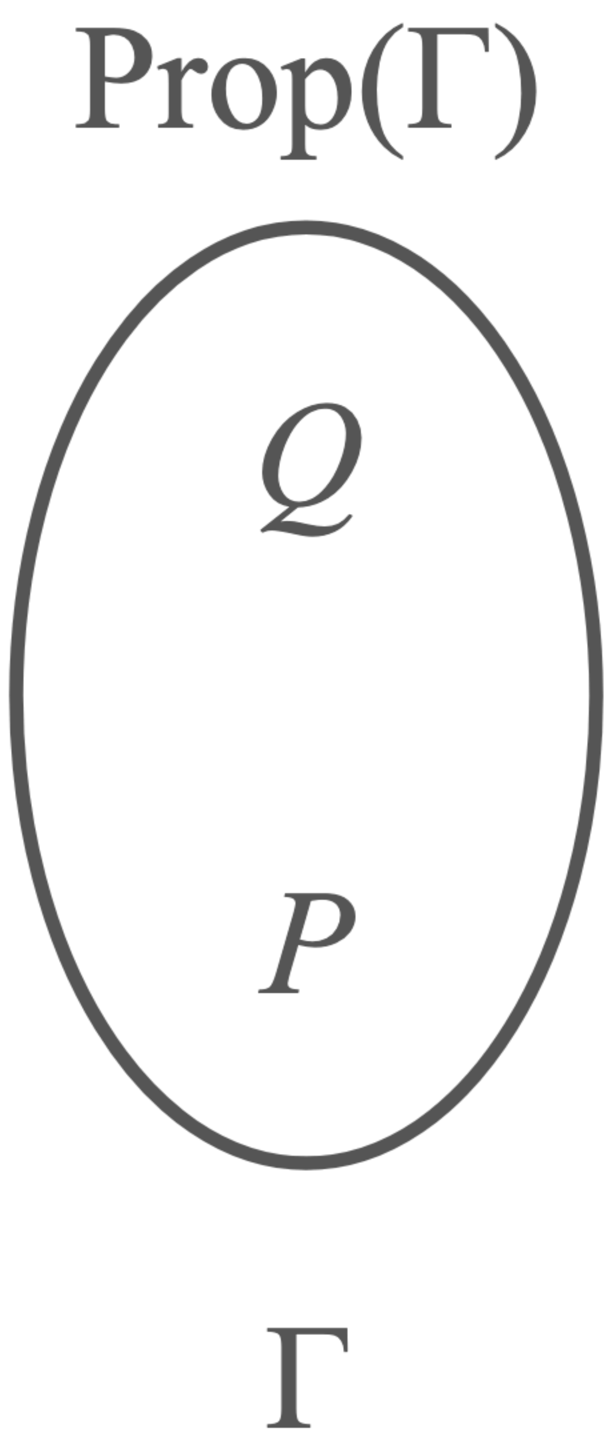

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$$\mathrm{Prop}(\Gamma)$$

$$Q$$

$$\rotatebox{90}{$\leq$}$$

$$P$$

$$\Gamma$$

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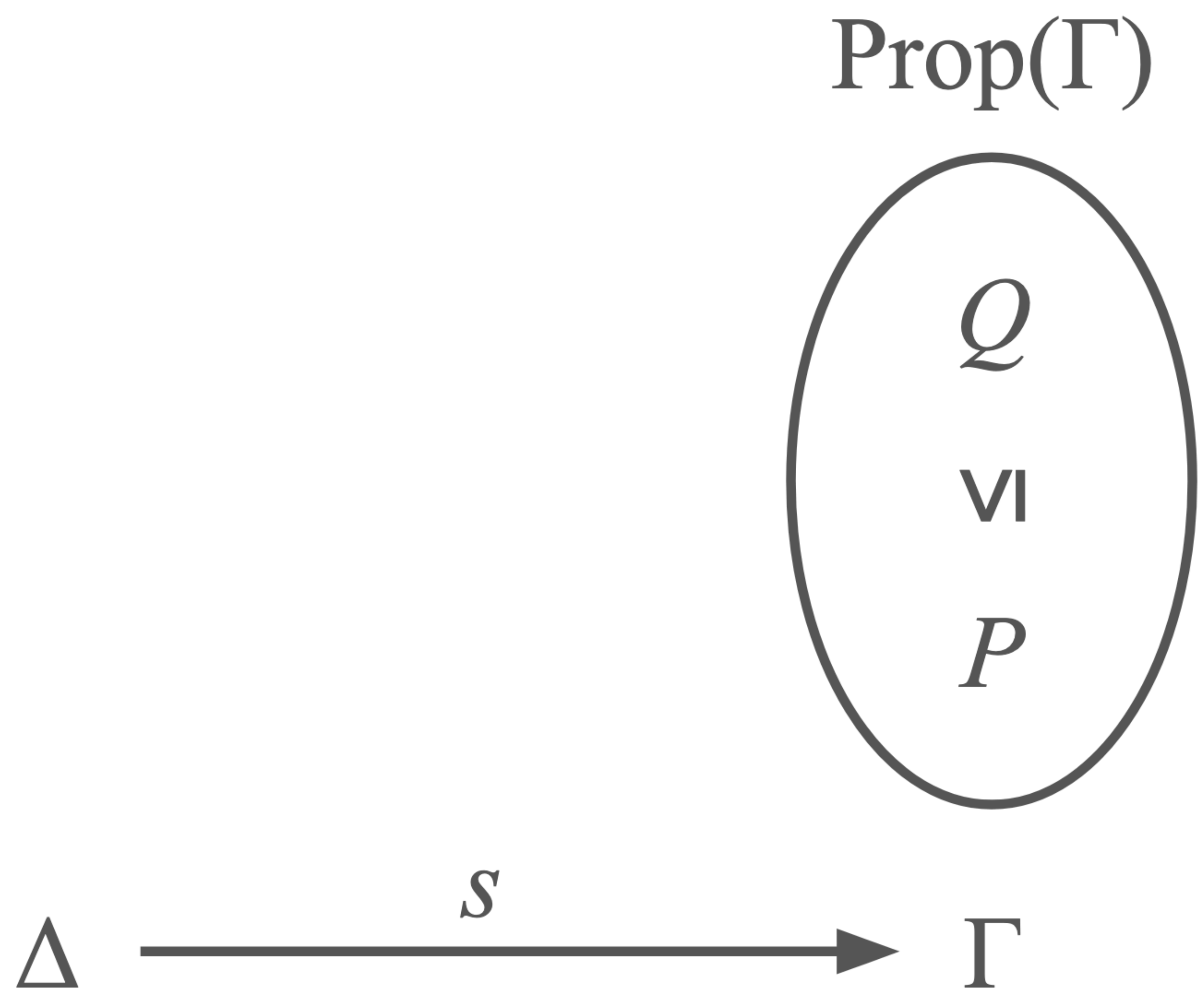

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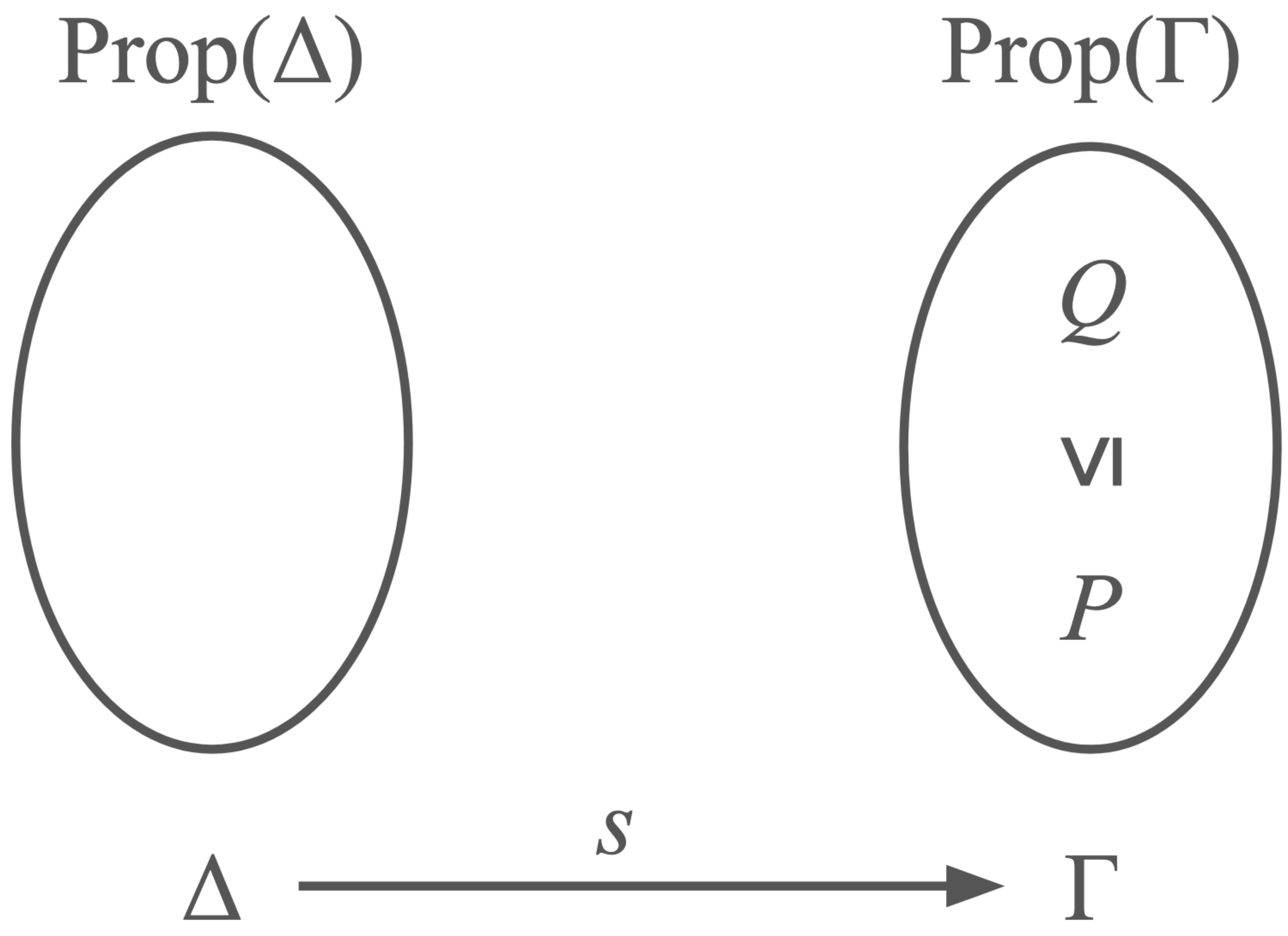

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$$\begin{tikzcd}
\mathrm{Prop}(\Delta) & & \mathrm{Prop}(\Gamma) \\
Q[s] & & Q \\
\rotatebox{90}{$\leq$} & & \rotatebox{90}{$\leq$} \arrow[ll, "{-[s]}"'] \\
P[s] & & P \\
\Delta \arrow[rr, "s"] & & \Gamma
\end{tikzcd}$$

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- Fibered alternative: find a category E and functor $p : E \longrightarrow \mathrm{Set}$ whose fibers are Prop (i.e., $p^{-1}(\Gamma) = \mathrm{Prop}(\Gamma)$ for all Γ).

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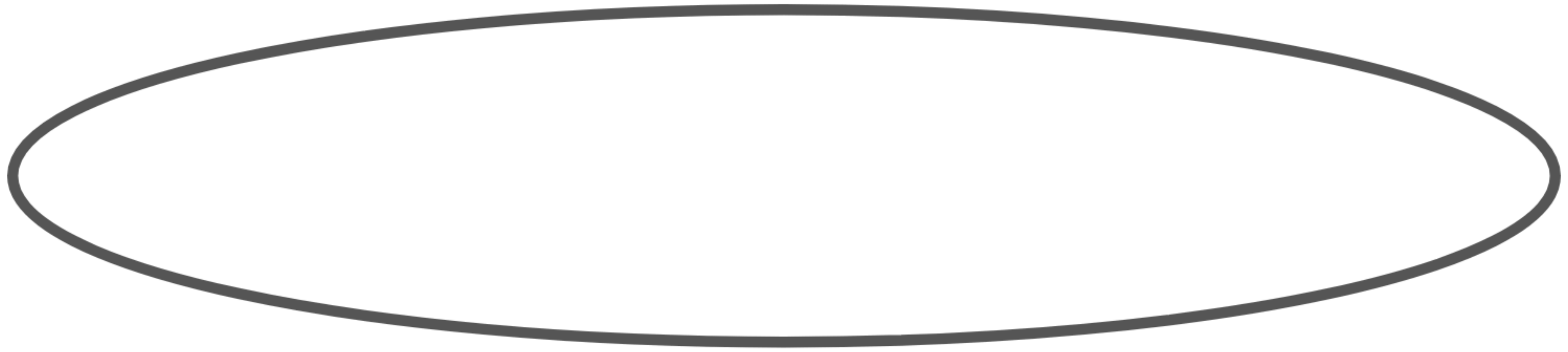
E



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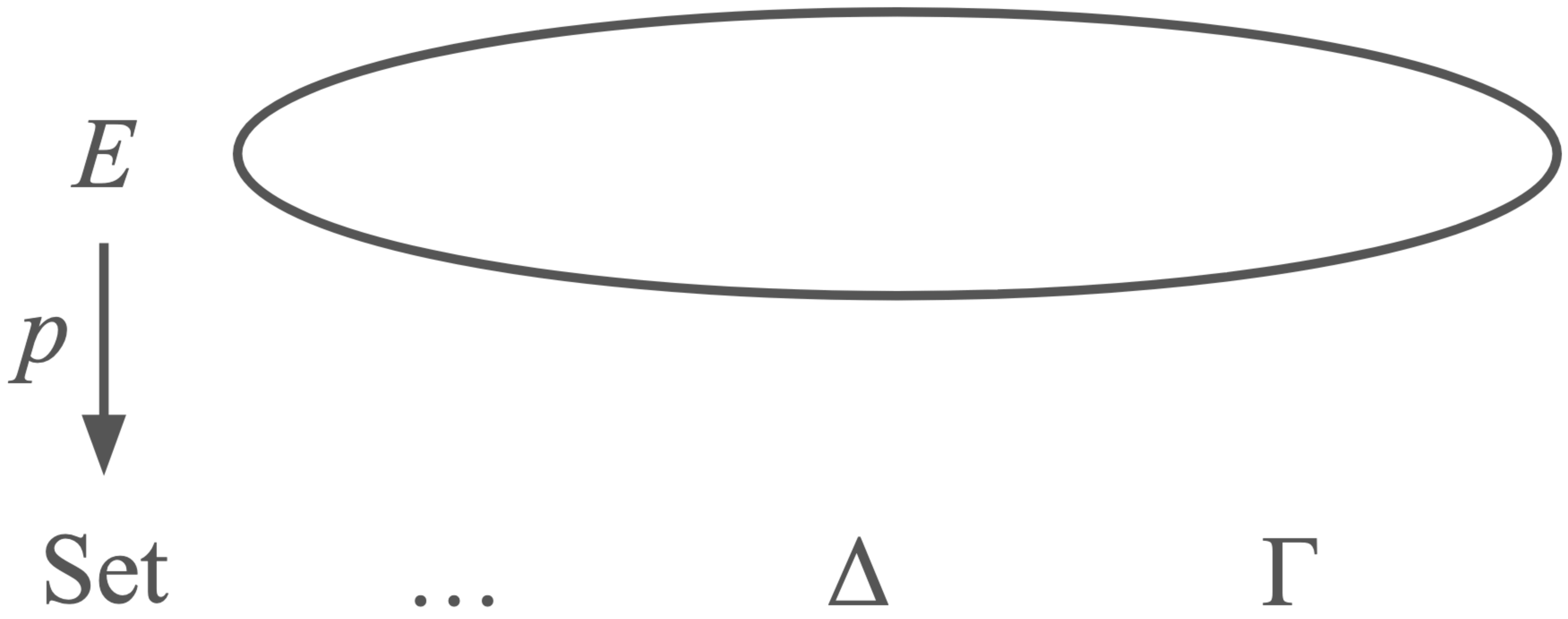

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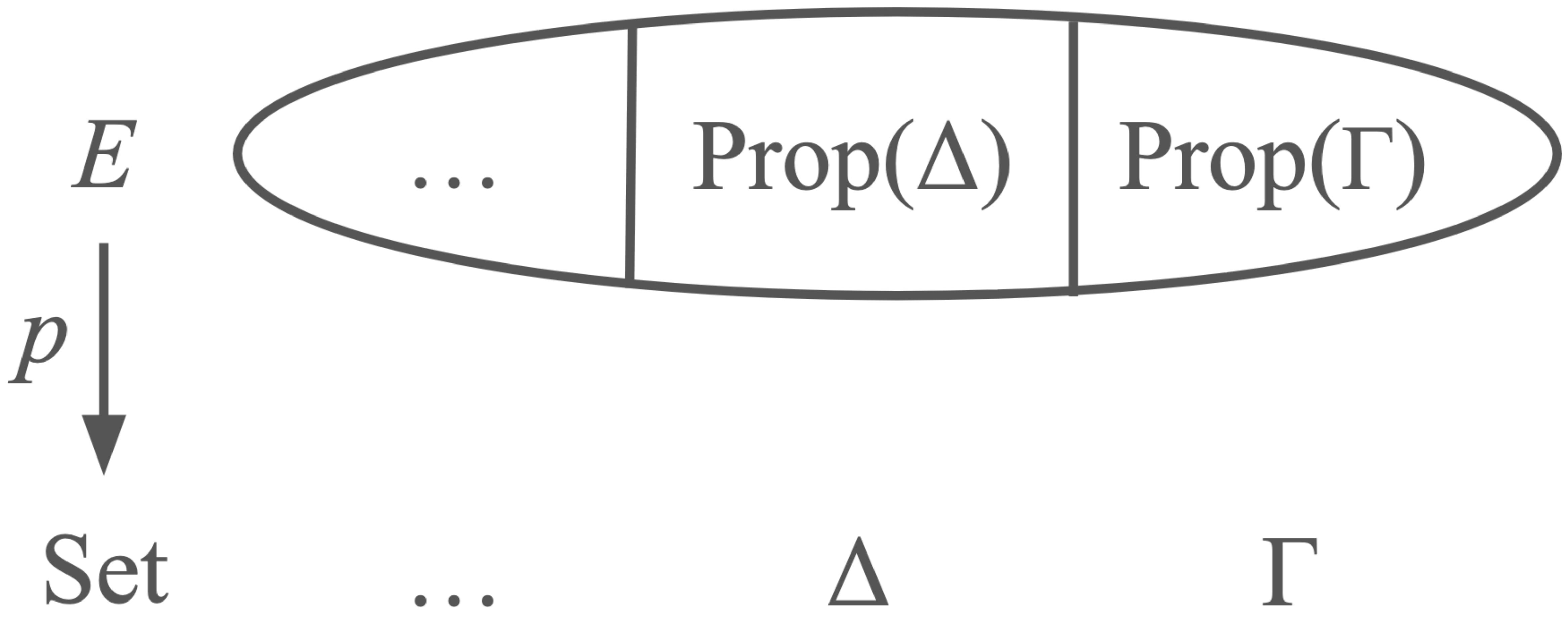

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- Such a functor $\begin{array}{c} E \\ p\Big\downarrow \\ \mathrm{Set} \end{array}$ is called a *fibration* with *total category* E.

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- Liftings give wps with the usual proof rules for ret, $>\!\!>\!=$.

Generalizing to substructural wp

What to do about the frame rule?

$$\frac{\{\, P \,\}\; e \;\{\, x.\, Q \,\}}{\{\, P \star F \,\}\; e \;\{\, x.\, Q \star F \,\}}$$

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- Observation: the frame rule is equivalent to

$$F \star \mathrm{wp}(e, x.\ Q) \vdash \mathrm{wp}(e, x.\ F \star Q)$$

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- Observation: the frame rule is equivalent to

$$F \star \mathrm{wp}(e, x.\ Q) \vdash \mathrm{wp}(e, x.\ F \star Q)$$

- This has the form of a *monad strength*:

$$X \otimes T(Y) \longrightarrow T(X \otimes Y)$$

The role of strength

$$\frac{\Gamma \vdash M : A \qquad \Delta, x : A \vdash N : B}{\Delta, \Gamma \vdash (\mathsf{let}\ \mathrm{x} = M\ \mathsf{in}\ N) : B}$$

The role of strength

$$\frac{\Gamma \xrightarrow{f} T(A) \qquad \Delta, x : A \vdash N : B}{\Delta, \Gamma \vdash (\mathrm{let}\ \mathrm{x} = M\ \mathrm{in}\ N) : B}$$

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$$\frac{\Gamma \xrightarrow{f} T(A) \qquad \Delta \otimes A \xrightarrow{g} T(B)}{\Delta, \Gamma \vdash (\text{let } \mathrm{x} = M \text{ in } N) : B}$$

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$$\frac{\Gamma \xrightarrow{f} T(A) \qquad \Delta \otimes A \xrightarrow{g} T(B)}{\Delta \otimes \Gamma \qquad\qquad\qquad T(B)}$$

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$$\begin{tikzcd}
\Delta \otimes \Gamma \arrow[d, "\Delta \otimes f"'] & & T(B) \\
\Delta \otimes T(B) & &
\end{tikzcd}$$

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$$\frac{\Gamma \xrightarrow{f} T(A) \qquad \Delta \otimes A \xrightarrow{g} T(B)}{\begin{tikzcd} \Delta \otimes \Gamma \arrow[d, "\Delta \otimes f"'] & & T(B) \\ \Delta \otimes T(B) \arrow[rr, "\sigma"] & & T(\Delta \otimes B) \end{tikzcd}}$$

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Substructural wp as a strong monad lifting

$$
\begin{tikzcd}
E \arrow[loop right, "{(T',\eta',\mu')}"] \arrow[d, "p"] \\
\mathrm{Set} \arrow[loop right, "{(T,\eta,\mu)}"]
\end{tikzcd}
$$

Substructural wp as a strong monad lifting

$$\begin{tikzcd} E \arrow[loop right, "{(T',\eta',\mu',\sigma') \text{ strong w.r.t. } (E,\otimes,I)}"] \arrow[d, "p"] \\ \mathrm{Set} \arrow[loop right, "{(T,\eta,\mu,\sigma) \text{ strong w.r.t. } (\mathrm{Set},\times,1)}"] \end{tikzcd}$$

Putting it all together

- A bunched monad lifting consists of:
  - A strong monad (T,η,μ,σ) on a category B with $\times, 1$
  - A strong monad (T',η',μ',σ') on a SMC $(E,\otimes,I)$ with $\times, 1$
  - A monoidal fibration $\begin{array}{c} E \\ \big\downarrow p \\ B \end{array}$ that sends (T',η',μ',σ') to (T,η,μ,σ)

Proposition

- Every bunched monad lifting T' of T models
  - conjunctive predicate BI, plus
  - a wp modality with the usual rules for ret, $>>=$, frame,

and wp commutes with subst. if T' is a fibered functor.

Example: vanilla separation logic

- Let $T : \mathrm{Set} \to \mathrm{Set}$ be the state monad $T(X) = \mathrm{Heap} \Rightarrow X \times \mathrm{Heap}$.

- Let E be the category of separation logic predicates:
  - Objects are pairs (X,P) where $P : X \to \mathrm{Heap} \to \mathrm{Prop}$
  - Morphisms $(X,P) \to (Y,Q)$ are functions $f : X \to Y$ such that $P(x) \subseteq Q(f(x))$

- Let $p : E \to \mathrm{Set}$ be the fibration $p(X,P) = X$.

Example: vanilla separation logic

- The monad T lifts along p to the monad

$$T' : E \longrightarrow E$$
$$T'(X,P) = (T(X),Q)$$

where $Q(c) = \{\ h \mid \forall f.\ f \uplus h$ defined $\Longrightarrow$
$\exists\, h'.\ c(f \uplus h) = (f \uplus h', x)$ and
$P(x)(h')\ \}$

A general construction

- Bunched monad liftings are stable under pullback:

$$
\begin{tikzcd}
E \arrow[loop right, "T'"] \arrow[d, "p"] \\
B \arrow[loop right, "T"]
\end{tikzcd}
$$

A general construction

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$$\begin{tikzcd}
 & E \arrow[d, "p"] \arrow[loop right, "T'"] \\
A \arrow[r, "f"'] \arrow[loop left, "S"] & B \arrow[loop right, "T"]
\end{tikzcd}$$

A general construction

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$$
\begin{tikzcd}
D \arrow[r, "g"] \arrow[d, "q"'] \arrow[loop left, "S'"] & E \arrow[d, "p"] \arrow[loop right, "T'"] \\
A \arrow[r, "f"'] \arrow[loop left, "S"] & B \arrow[loop right, "T"]
\end{tikzcd}
$$

Future and ongoing work

- Algebraic presentations of wp

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$$(\ell \mapsto \text{-}) \star (\forall\, v.\ (\ell \mapsto v) \mathrel{-\!\!\star} \mathrm{wp}(k(v),\ x.\ Q)) \vdash \mathrm{wp}(\mathrm{get}(\ell) \mathbin{>\!\!>\!\!=} k,\ x.\ Q)$$

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$$L \quad \otimes \quad (V \quad \multimap \quad T(X)) \quad \longrightarrow \quad T(X)$$

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$$(L \otimes V) \otimes (\quad I \qquad \multimap \qquad T(X) \quad) \quad \longrightarrow \qquad\qquad T(X)$$

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- Monad transformers for bunched liftings

wp for T

$$\begin{tikzcd} E \arrow[loop right, "T'"] \arrow[d] \\ B \arrow[loop right, "T"] \end{tikzcd}$$

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wp for $T + \mathrm{Err}$

$$\begin{tikzcd} E \arrow[loop right, "T' + \mathrm{Err}'"] \arrow[d] \\ B \arrow[loop right, "T + \mathrm{Err}"] \end{tikzcd}$$

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wp for T

$$
\begin{tikzcd}
E \arrow[loop right, "T'"] \arrow[d] \\
B \arrow[loop right, "T"]
\end{tikzcd}
$$

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Step-indexed wp for $\mu(T \circ \triangleright)$

$$\begin{tikzcd}
{[\mathbb{N}^{op}; E]} \arrow[loop right, "\mu(T' \circ \triangleright)"] \arrow[d] \\
{[\mathbb{N}^{op}; B]} \arrow[loop right, "\mu(T \circ \triangleright)"]
\end{tikzcd}$$

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> **BI-Hyperdoctrines, Higher-Order Separation Logic, and Abstraction**
>
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> IT University of Copenhagen

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