

New foundations for probabilistic separation logic

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<https://john-ml.github.io/lilac.pdf>

How to modularly verify probabilistic programs?

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- How to perform verification and inference at scale?

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$$\begin{array}{ccc} \begin{array}{c} h_1 \\ \boxed{} \boxed{} \boxed{} \end{array} \uplus \begin{array}{c} h_2 \\ \boxed{} \boxed{} \end{array} \models P * Q & \text{if} & \begin{array}{c} h_1 \\ \boxed{} \boxed{} \boxed{} \end{array} \models P \\ & & \begin{array}{c} h_2 \\ \boxed{} \boxed{} \end{array} \models Q \end{array}$$

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- Can we adapt this to probabilistic setting?

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Traditional heap-separation is not enough

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A Separation Logic for Concurrent Randomized Programs

JOSEPH TASSAROTTI, Carnegie Mellon University, USA
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POPL'19

Quantitative Separation Logic

A Logic for Reasoning about Probabilistic Pointer Programs

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- With randomness, there's a second resource to account for: probability.
- Heap-separation doesn't support modular reasoning about this resource.

Separation as statistical independence

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A Probabilistic Separation Logic

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A Separation Logic for Negative Dependence

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A Bunched Logic for Conditional Independence

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- Propositions modeled by distributions on stores

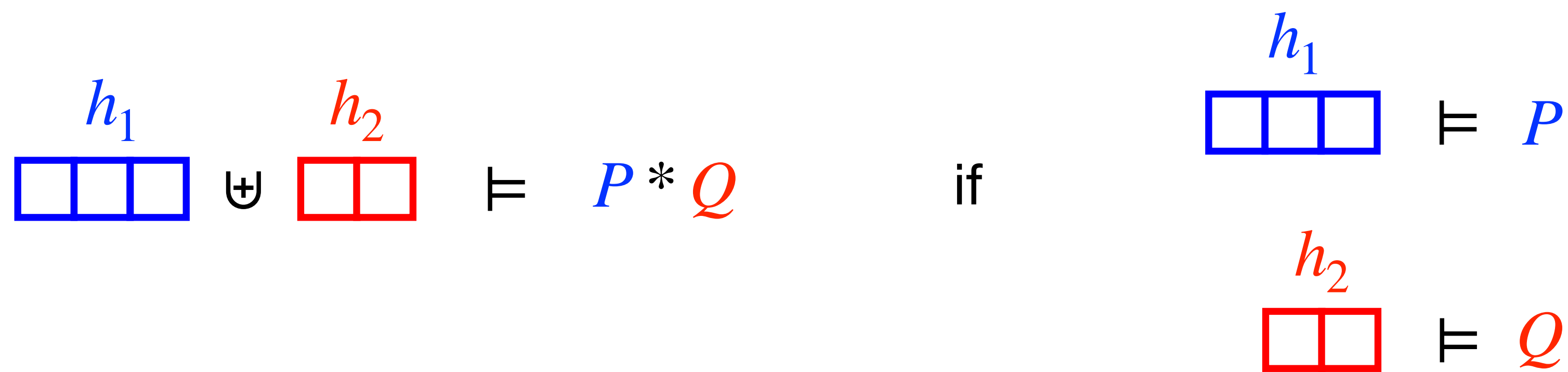
A new notion of separation

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- Propositions modeled by *probability spaces*

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A new notion of separation

- Propositions modeled by *probability spaces*

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"independent combination"

What does this buy us?

- Standard frame rule
- Continuous variables & equational reasoning
- Conditioning modality $D_{x \leftarrow X}$
 - Conditional independence: $D_{x \leftarrow X}(Y * Z)$
 - Other connectives also have intuitive "conditional" readings
- Verified challenging weighted sampling algorithm
- Full paper: <https://john-ml.github.io/lilac.pdf>